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# Dual Symmetry in Maxwell equations

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## Abstract

Dual symmetry is a remarkable property of Maxwell's equations in vacuum and a direct group-theoretical result of the representations of the Lorentz group for spin 1 massless particles. Unlike in Dirac theory they the two representations are physically equivalent for spin 1. The consequence is the existence of a "dual" symmetry between electric and magnetic fields according to  $\mathbf{E}$  to  $\mathbf{B}$  and  $\mathbf{B}$  to  $-\mathbf{E}$  that is immediately broken when matter is added via the macroscopic constants  $\epsilon$  and  $\mu$ . In quantum theory, phase and canonical momentum are connected by gauge transformations. An electric dipole  $\mathbf{d}$  in a homogeneous magnetic field has a quantum phase  $\mathbf{B} \times \mathbf{d}$  that is connected to the topological Aharonov-Bohm phase. The Aharonov-Casher topological quantum phase of a magnetic dipole  $\mathbf{m}$  in an electric field is proportional to  $\mathbf{m} \times \mathbf{E}$  and its existence can be put in the light of this controversy, illustrating that it is often better defined when reformulated microscopically. The controversy pops up again when matter is coupled to the electromagnetic quantum vacuum. An Aharonov-Casher type force emerges when we couple a highly excited Rydberg atom to an electric field, mediated by virtual highly energetic photons.

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